

nucl-th/9911034  
 DOE/ER/40561-74-INT99  
 DUKE-TH-99-198  
 NT@UW-99-61  
 TUM-T39-99-23  
 9th November 1999  
 Final, revised version  
 12th April 2000

# Higher Partial Waves in an Effective Field Theory Approach to $nd$ Scattering

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## Abstract

The phase shifts for the higher partial waves ( $l \geq 1$ ) in the spin quartet and doublet channel of  $nd$  scattering at centre-of-mass energies up to 15 MeV are presented at next-to-leading and next-to-next-to-leading order in an effective field theory in which pions are integrated out. As available, the results agree with both phase shift analyses and potential model calculations.

Suggested PACS numbers: 11.80.J, 21.30.-x, 21.45.+v, 25.10.+s, 25.40.Dn

Suggested Keywords: effective field theory, nucleon-deuteron scattering, three-body systems

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# 1 Introduction

In the past few years, the Effective Field Theory approach [1] has proven a useful tool to describe the nuclear few body problem. Like the related low energy theories of QCD, Chiral Perturbation Theory [2, 3] and Heavy Baryon Chiral Perturbation Theory [4, 5] in the zero and one nucleon sector respectively, it uses the fact that QCD exhibits a separation of scales: The scales associated with confinement and chiral symmetry are so high that it is natural to formulate QCD at low energies typical for nuclear processes only in terms of the degrees of freedom which emerge after quarks and gluons are confined in bound states. Below the pion cut, even pions are absent and nucleons emerge as the only effective degrees of freedom. A low energy expansion of QCD is then an expansion in powers of the typical momentum of the process  $Q$  in units of the high energy scale  $\Lambda$  at which the theory breaks down. This theory both with and without explicit pions was put to extensive tests in the two-nucleon sector in recent years [6]-[13], starting with the pioneering work of Weinberg who suggested the usefulness of EFT's in nuclear physics [14]. It allowed for the first time for predictions which are systematic, rigorous and model-independent (i.e. independent of assumptions about the underlying non-perturbative QCD dynamics). In most cases investigated, the experimental agreement is within the estimated theoretical uncertainties, and in some cases, previously unknown counterterms describing short distance physics could be determined. Although in general process dependent, the expansion parameter is found to be of the order of  $\frac{1}{3}$  in most applications, so that NLO calculations can be expected to be accurate to about 10%, and NNLO calculations to about 4%. The theory with pions integrated out was recently pushed to very high orders in the two-nucleon sector [15] where accuracies of the order of 1% were obtained. It can be viewed as a systematisation of the Effective Range Theory (ERT) with the inclusion of relativistic and short distance effects not found in traditional ERT treatments.

The simplest problem in the three-nucleon sector is  $nd$  scattering in the spin-quartet S wave: The absence of Coulomb interactions ensures that only properties of the strong interactions are probed, and the Pauli principle guarantees that the three nucleons cannot occupy the same point in space, making the effect of three-body forces very small. The phase shifts in this channel were shown to be well reproduced with and without pions both below and above the deuteron breakup point up to momenta of 400 MeV in the centre of mass frame [16]. The explicit inclusion of pions was shown not to improve the accuracy even at those relatively high momenta. In the S wave, spin-doublet channel (the triton channel) the situation is more complicated. An unusual renormalisation of the three-body force makes it large and as important as the leading two-body forces. Still, by using the triton binding energy to fix the value of the three-body force, a reasonable description of the phase shift in the doublet channel can be obtained [17]. In this article, we apply the EFT with pions integrated out to the higher partial waves in the  $nd$  system, above and below the deuteron breakup point. All parameters are determined from  $NN$  scattering, and three-body forces do not enter at the order we calculate, allowing one to determine the range of validity of the pion-less EFT without a detailed analysis of the fitting procedure. Since some problems depending critically on some of these channels remain unsolved, like

the  $A_y$  puzzle, the calculations presented are a testing ground for the applicability of the EFT approach to these questions.

In Sect. 2, we present a sketch of the EFT in the many nucleon sector with pions integrated out. The application to  $nd$  scattering in Sect. 3 is followed by a discussion of our results in the last Section.

## 2 Formalism

### 2.1 Lagrangean and Power Counting

We now sketch the theory underlying our calculations, with more details to be found in [16]. Three main ingredients enter in the formulation of an EFT: the Lagrangean, the power counting and a regularisation scheme.

After identifying the relevant degrees of freedom in a low energy theory of QCD as the nucleons, one writes down the most general Lagrangean compliant with the symmetries of QCD. Since a theory without explicit pions is expected to break down approximately at the pion cut,  $\Lambda \approx 100$  MeV, the nucleons can be treated as non-relativistic particles with Lorentz invariance restored by higher order terms. Without pions, only contact interactions are allowed between nucleons. The first few terms in the most general, iso-spin invariant Lagrangean are therefore

$$\begin{aligned} \mathcal{L}_{NN} = & N^\dagger (i\partial_0 + \frac{\vec{\partial}^2}{2M} - \frac{\partial_0^2}{2M}) N - \\ & - C_{0d} (N^T P_d^i N)^\dagger (N^T P_d^i N) - \\ & - C_{0t} (N^T P_t^A N)^\dagger (N^T P_t^A N) + \\ & + \frac{C_{2d}}{8} \left[ (N^T P_d^i N)^\dagger (N^T P_d^i (\vec{\partial} - \overleftarrow{\partial})^2 N) + \text{H.c.} \right] + \\ & + \frac{C_{2t}}{8} \left[ (N^T P_t^A N)^\dagger (N^T P_t^A (\vec{\partial} - \overleftarrow{\partial})^2 N) + \text{H.c.} \right] + \\ & + \dots \ , \end{aligned} \tag{2.1}$$

where  $N = \begin{pmatrix} p \\ n \end{pmatrix}$  is the nucleon doublet of two-component spinors and the sub-scripts  $d$  and  $t$  denote the  $^3S_1$  and  $^1S_0$  channel of  $NN$  scattering. For example,  $P_d^i$  and  $P_t^A$  are the projectors onto the iso-scalar vector and scalar iso-vector channels,

$$(P_d^i)_{a\alpha}^{b\beta} = \frac{1}{\sqrt{8}} (\sigma_2 \sigma^i)_\alpha^\beta (\tau_2)_a^b \ , \quad (P_t^A)_{a\alpha}^{b\beta} = \frac{1}{\sqrt{8}} (\sigma_2)_\alpha^\beta (\tau_2 \tau^A)_a^b \ , \tag{2.2}$$

with  $\sigma$  ( $\tau$ ) the Pauli matrices acting in spin (iso-spin) space. A three-nucleon force appears at low orders only in the doublet S channel [17]. The last of the nucleon kinetic energy terms in the Lagrangean restores Lorentz invariance [15]. The coefficients  $C_i$  encode all short distance physics – like pion and  $\omega$  exchanges, quarks and gluons, and resonances like the  $\Delta$  – as the strengths of potentials built out of derivatives of  $\delta$  functions. In principle,

these constants could be derived by solving QCD or from models of short distance physics, but the most common and practical way is to determine them from experiment.

Since the Lagrangean (2.1) consists of infinitely many terms only restricted by symmetry, predictive power is ensured only by the second ingredient of an EFT: a power counting scheme, i.e. a way to determine at which order in a momentum expansion different contributions will appear, keeping only and all the terms up to a given order in calculations. The dimensionless, small parameter on which the expansion is based is the typical momentum  $Q$  of the process in units of the scale  $\Lambda$  at which the theory is expected to break down, e.g. in a pion-less theory in units of the pion mass. Assuming that all contributions are of natural size, i.e. ordered by powers of  $Q$ , the systematic power counting ensures that the sum of all terms left out when calculating to a certain order in  $Q$  is smaller than the last order retained. This way, an EFT allows for an error estimate of the accuracy of the final result.

In the two-nucleon sector, finding such a power counting scheme is complicated by the fact that the scattering lengths of both the  $^1S_0$  and  $^3S_1$  channel in  $NN$  scattering are unnaturally large ( $1/a_t = -8.3$  MeV,  $1/a_d = 36$  MeV) because of the existence of a shallow virtual and real bound state, the latter being the deuteron with a binding energy  $B = 2.225$  MeV and hence a typical binding momentum  $\gamma_d = \sqrt{MB} \simeq 46$  MeV. This lies well below the expected breakdown scale of the EFT and suggests that the system is close to a non-trivial fixed point<sup>1</sup>. Due to this fine tuning, the naïve low momentum expansion has a very limited range of validity ( $Q \lesssim 1/a_t, Q \lesssim 1/a_d$ ). The alternative is to expand in powers of the small parameter  $Q/\Lambda \ll 1$  but to keep the full dependence on  $Qa_t, Qa_d \sim 1$ . This is clearly necessary if one is to be able to describe bound states within the effective theory. Some interactions have therefore to be treated non-perturbatively in order to accommodate these bound states. In the effective theory, the fine tuning present in the two-body interactions arises as cancellation between loop and counterterm contributions. This makes it tricky to do the power counting in a naïve approach since it cannot be applied diagram by diagram, but only to whole classes of diagrams which are furthermore not always easily identified [19]. A convenient way of dealing with this problem was suggested in [20]. It consists in using dimensional regularisation and a new subtraction scheme (Power Divergence Subtraction, PDS) in which not only the poles Feynman graphs exhibit in 4 dimensions are subtracted but also the poles in 3 dimensions. This regularisation procedure is chosen to explicitly preserve the systematic power counting as well as all symmetries at each order in every step of the calculation. If one chooses the arbitrary scale  $\mu$  arising in the subtraction to be of the order  $\mu \sim Q \sim 1/a_t \sim 1/a_d$ , one finds

$$\begin{aligned} C_0^{(-1)} &\sim \frac{1}{MQ} \ , \\ C_0^{(0)} &\sim \frac{1}{M\Lambda} \ , \end{aligned} \tag{2.3}$$

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<sup>1</sup>See e.g. [18] for an analysis of this short distance fine tuning problem in terms of critical points of the renormalisation group.

$$C_2 \sim \frac{1}{M\Lambda Q^2}$$

for the short distance coefficients of both the  $^3S_1$  and  $^1S_0$  channel. Here, as in the following, we split the coefficients of the leading four point interactions,  $C_0$ , into a leading ( $C_0^{(-1)}$ ) and a sub-leading piece ( $C_0^{(0)}$ ), where the super-script in parenthesis denotes the scaling of the coefficient with  $Q$ .

Using (2.3) one can estimate the contribution of any given diagram. As will be demonstrated in the next sub-section, it is found that for nucleon-nucleon scattering, the leading order contribution is given by an infinite number of diagrams forming a bubble chain, with the vertices being the ones proportional to  $C_0$ . Higher order corrections are perturbative and given by one or more insertions of higher derivative operators. In the three-body sector discussed in this paper, even the leading order calculation is too complex for a fully analytical solution. Still, the equations that need to be solved are computationally trivial and can furthermore be improved systematically by higher order corrections that involve only integrations, as opposed to many-dimensional integral equations arising in other approaches.

Following a previous paper on quartet S wave scattering in the  $nd$  system [16], we choose a Lagrangean which is equivalent to (2.1) but contains two auxiliary fields  $d^i$  and  $t^A$  with the quantum numbers of the deuteron and of a di-baryon field in the  $^1S_0$  channel of  $NN$  scattering respectively, such that the four nucleon interactions are removed:

$$\begin{aligned} \mathcal{L}_{Nd} = & N^\dagger (i\partial_0 + \frac{\vec{\partial}^2}{2M} - \frac{\partial_0^2}{2M}) N + \\ & -d^{i\dagger} \left[ (i\partial_0 + \frac{\vec{\partial}^2}{4M}) + \Delta_d^{(-1)} + \Delta_d^{(0)} \right] d^i + y_d [d^{i\dagger} (N^T P_d^i N) + \text{H.c.}] + \\ & -t^{A\dagger} \left[ (i\partial_0 + \frac{\vec{\partial}^2}{4M}) + \Delta_t^{(-1)} + \Delta_t^{(0)} \right] t^A + y_t [t^{A\dagger} (N^T P_t^A N) + \text{H.c.}] + \dots \end{aligned} \quad (2.4)$$

It was demonstrated in [16] that this Lagrangean is more convenient for the numerical investigations necessary in the three-body system. Analogously to the  $C_0$ 's, we split the  $\Delta$ 's into leading ( $\Delta^{(-1)}$ ) and sub-leading pieces ( $\Delta^{(0)}$ ). Performing the Gaussian integration over the auxiliary fields  $d^i$  and  $t^A$  in the path integral followed by a field re-definition in order to eliminate the terms containing time derivatives, we see that the two Lagrangeans (2.1) and (2.4) are indeed equivalent up to higher order terms when one identifies

$$\begin{aligned} \Delta^{(-1)} &= -\frac{C_0^{(-1)}}{MC_2} , \\ \Delta^{(0)} &= \frac{C_0^{(0)}}{MC_2} , \\ y^2 &= \frac{(C_0^{(-1)})^2}{MC_2} \end{aligned} \quad (2.5)$$

for each of the two S wave channels separately. The “wrong” sign of the kinetic energy terms of the auxiliary fields therefore does not spoil unitarity. The new coefficients scale from (2.3) and (2.5) in each channel as

$$\begin{aligned}\Delta^{(-1)} &\sim \frac{Q\Lambda}{M} , \\ \Delta^{(0)} &\sim \frac{Q^2}{M} , \\ y^2 &\sim \frac{\Lambda}{M^2} .\end{aligned}\tag{2.6}$$

## 2.2 The Two-Nucleon System to NNLO

It is useful to consider the solution to the two-body problem before turning to the three-body problem. As the  $^3S_1$  and  $^1S_0$  channels do not differ in their power counting, we concentrate on the deuteron channel. The considerations in the  $^1S_0$  channel are analogous. More details about the EFT without pions in the triplet channel can be found in [15].

Any diagram can be estimated by scaling all momenta by a factor of  $Q$  and all energies by a factor of  $Q^2/M$ . The remaining integral includes no dimensions and is taken to be of the order  $Q^0$  and of natural size. This scaling implies the rule that nucleon propagators contribute one power of  $M/Q^2$  and each loop a power of  $Q^5/M$ . Relativistic corrections to the kinetic energy of the nucleon scale like  $\frac{Q^4}{M^3}$  and hence only enter as insertions into the non-relativistic nucleon propagator at NNLO in  $Q$ , suppressed by additional powers of  $1/M$ . The vertices provide powers of  $Q$  according to (2.6), implying that the deuteron kinetic energy term is sub-leading compared to the  $\Delta_d^{(-1)}$  term. Thus, the bare deuteron propagator is just the constant  $-i/\Delta_d^{(-1)}$ . Using these rules, there is an infinite number of diagrams contributing at leading order to the deuteron propagator, as shown in Fig. 1, each one of the order  $1/(MQ)$ . The linear divergence in each of the bubble diagrams shown in

Figure 1: *The deuteron propagator at leading order from the Lagrangean (2.4). The thick solid line denotes the bare propagator  $\frac{-i}{\Delta_d^{(-1)}}$ , the double line its dressed counterpart.*

Fig. 1 does not show in dimensional regularisation as a pole in 4 dimensions, but it does appear as a pole in 3 dimensions which we subtract following the PDS scheme.

The full leading order propagator  $i\Delta_d^{ij}(p)$  of the deuteron field consists hence of the geometric series shown in Fig. 1,

$$i\Delta_d^{ij}(p) = -\frac{4\pi i}{My_d^2} \frac{\delta^{ij}}{\frac{4\pi\Delta_d^{(-1)}}{My_d^2} - \mu + \sqrt{\frac{\vec{p}^2}{4} - Mp_0 - i\varepsilon}} ,\tag{2.7}$$

where  $\mu$  is the arbitrary renormalisation scale introduced by the PDS scheme, showing that  $\Delta_d^{(-1)}/y_d^2$  is renormalisation group dependent, too. Physical observables like amplitudes are of course independent of the choice of  $\mu$ , as will be demonstrated now. We determine the free parameters by demanding that the deuteron propagator has its pole at the physical deuteron pole position as

$$-\frac{y_d^2}{\Delta_d^{(-1)}} = \frac{4\pi}{M} \frac{1}{\gamma_d - \mu} = C_{0d}^{(-1)} . \quad (2.8)$$

Since  $\gamma_d \sim Q$ , the choice  $\mu \sim Q$  makes the coefficient  $C_{0d}^{(-1)}$  indeed scale as required in the power counting scheme (2.3). The deuteron and di-baryon propagators are hence in terms of physical quantities at leading order given by

$$\begin{aligned} i\Delta_d^{ij}(p) &= i\delta^{ij} \Delta_d(p) = \frac{4\pi i}{My_d^2} \frac{\delta^{ij}}{\gamma_d - \sqrt{\frac{\vec{p}^2}{4} - Mp_0 - i\varepsilon}} , \\ i\Delta_t^{AB}(p) &= i\delta^{AB} \Delta_t(p) = \frac{4\pi i}{My_t^2} \frac{\delta^{AB}}{\gamma_t - \sqrt{\frac{\vec{p}^2}{4} - Mp_0 - i\varepsilon}} . \end{aligned} \quad (2.9)$$

The typical momentum  $\gamma_t = 1/a_t$  of the virtual bound state is extracted from the scattering length in this channel. Each deuteron or di-baryon propagator is accompanied by a power of  $1/(My^2Q) = M/(\Lambda Q)$ .

The kinetic energy insertion and the mass insertion proportional to  $\Delta_d^{(0)}$  are suppressed by one power of  $Q$  according to the scaling properties (2.6) and hence enter at NLO. We choose to keep the  $\Delta^{(-1)}$ 's to be the same as at LO, so that (2.8) is still valid at NLO and higher order calculations will not necessitate a re-fitting of lower order coefficients [21]. The remaining pieces,  $\Delta^{(0)}$ , are then parametrically smaller (as indicated by (2.6)) and are included perturbatively. Two conditions per partial wave are now necessary to fix the new constants at NLO. One condition is that the pole position of the bound state does not change, Fig. 2:

$$\Delta_d^{(0)} = \frac{\gamma_d^2}{M} \quad (2.10)$$

The second condition in the  $^3S_1$  channel can be that the deuteron pole has the correct residue  $Z_d$  [22],

$$\frac{8\pi\gamma_d}{M^2y_d^2} = Z_d - 1 , \quad Z_d^{-1} = 1 - \gamma_d\rho_d , \quad (2.11)$$

where  $\rho_d$  is defined by the effective range expansion around the deuteron pole,

$$k \cot \delta = -\gamma_d + \frac{1}{2}\rho_d(k^2 + \gamma_d^2) + \dots , \quad (2.12)$$

for the nucleon-nucleon phase shifts. In the singlet channel, no real bound state exists whose properties have to be described, so we impose the condition that the effective range expansion

$$k \cot \delta = -\frac{1}{a_t} + \frac{1}{2}r_{0t}k^2 + \dots \quad (2.13)$$

$$\begin{aligned}
& \left[ \text{diagram: double line with a shaded circle in the middle} \right]^{-1} \Big|_{p_0 = -\frac{\gamma_d^2}{M}, \vec{p}=0} \stackrel{!}{=} 0 \quad : \\
& \left[ \text{diagram: double line} \right]^{-1} \Big|_{p_0 = -\frac{\gamma_d^2}{M}, \vec{p}=0} \stackrel{!}{=} 0 \quad , \quad \left[ \text{diagram: double line with a cross} + \text{diagram: double line with a dot} \right] \Big|_{p_0 = -\frac{\gamma_d^2}{M}, \vec{p}=0} \stackrel{!}{=} 0
\end{aligned}$$

Figure 2: The first condition on the coefficients  $\Delta_d^{(-1)}$ ,  $\Delta_d^{(0)}$  and  $y_d$ , (2.8/2.10). The dressed deuteron propagator has its pole at the physical binding energy at LO, and higher order corrections do not change its position. The cross denotes a kinetic energy insertion, the dot an insertion of  $\Delta_d^{(0)}$ .

is reproduced. Solving (2.10/2.11/2.13) with (2.8) leads to

$$\begin{aligned}
\Delta_d^{(-1)} &= \frac{2}{M} \frac{\mu - \gamma_d}{\rho_d} \quad , \quad \Delta_t^{(-1)} = \frac{2}{M} \frac{\mu - \gamma_t}{r_{0t}} \quad , \\
\Delta_d^{(0)} &= \frac{\gamma_d^2}{M} \quad , \quad \Delta_t^{(0)} = 0 \quad , \\
y_d^2 &= \frac{8\pi}{M^2} \frac{\gamma_d}{Z_d - 1} \quad , \quad y_t^2 = \frac{8\pi}{M^2} \frac{1}{r_{0t}} \quad .
\end{aligned} \tag{2.14}$$

In a NNLO calculation,  $C_2$  is split like  $C_0$ , with  $C_2^{(-2)}$  still given by the expression above,  $C_2^{(-1)}$  a NNLO correction parametrically small against  $C_2^{(-2)}$ . Likewise,  $C_0$  has now a NNLO component  $C_0^{(1)}$  which is again fixed by the condition that the deuteron or di-baryon pole position is unchanged. This way, higher order calculations do not necessitate a re-fitting of lower order coefficients [21]. In addition, four new interactions enter at NNLO [15]: The first is a term which induces SD mixing in the two-nucleon system and will be shown to be irrelevant for our problem. Relativistic corrections to the nucleon propagator,  $-\frac{\partial_0^2}{2M} \sim \frac{Q^4}{M^3}$ , enter at NNLO in the  $Q$  counting, but are suppressed by two more powers of  $M$  and hence can be neglected. Finally, there are  $NN$  interactions containing four derivatives. In the triplet channel, for example, a term

$$-\frac{1}{16} C_{4d} (N^T P_d^i (\vec{\partial} - \vec{\partial})^2 N)^\dagger (N^T P_d^i (\vec{\partial} - \vec{\partial})^2 N) \tag{2.15}$$

enters in the Lagrangean (2.1) which by Gaußian integration and field re-definition can be shown to be equivalent to a term in the modified Lagrangean (2.4) of the form

$$\frac{MC_{4d}}{C_{2d}^{(-2)}} d^{i\dagger} \left[ i\partial_0 + \frac{\vec{\partial}^2}{4M} \right]^2 d^i \quad . \tag{2.16}$$

However, it was shown in [15] that the renormalisation group equations determine the coefficient  $C_4$  through the lower order constants  $C_0$  and  $C_2$ , i.e. that the value of this



constant is determined already from the NLO parameters as [22]

$$C_{4d} = -\frac{\pi}{M} \frac{(Z_d - 1)^2}{\gamma_d^2(\mu - \gamma_d)^3} . \quad (2.17)$$

This is a generic feature, the leading piece of these higher derivative terms are determined by the LO and NLO coefficients. Their independent contribution start at N3LO but can be re-summed into a full deuteron (di-baryon) propagator

$$\begin{aligned} i\Delta_d^{ij}(p; \text{N3LO}) &= i\delta^{ij} \Delta_d(p; \text{N3LO}) = \\ &= -\frac{4\pi i}{My_d^2} \frac{\delta^{ij}}{-\gamma_d + \frac{1}{2}\rho_d(Mp_0 - \frac{\vec{p}^2}{4} + \gamma_d^2) + \sqrt{\frac{\vec{p}^2}{4} - Mp_0 - i\varepsilon}} , \\ i\Delta_t^{AB}(p; \text{N3LO}) &= i\delta^{AB} \Delta_t(p; \text{N3LO}) = \\ &= -\frac{4\pi i}{My_t^2} \frac{\delta^{AB}}{-\gamma_t + \frac{1}{2}r_{0t}(Mp_0 - \frac{\vec{p}^2}{4}) + \sqrt{\frac{\vec{p}^2}{4} - Mp_0 - i\varepsilon}} . \end{aligned} \quad (2.18)$$

### 3 The $nd$ System

#### 3.1 Quartet Channel

Turning to the  $nd$  system, we first consider the quartet channel as its treatment is analogous to the doublet channel without the difficulties of a coupled channel equation. For details, we refer again to the treatment of the quartet S wave in [16].

All three nucleon spins are aligned, so that only properties of the triplet  $NN$  system are probed. It has been shown in [16] that all diagrams in which three nucleons interact only via the LO deuteron-nucleon potential  $y_d$  are of the same, leading order  $\Lambda/(MQ^2)$ . Summing all “bubble-chain” sub-graphs of Fig. 1 into the deuteron propagator, one arrives at the integral equation pictorially represented in Fig. 3. To derive this equation for the

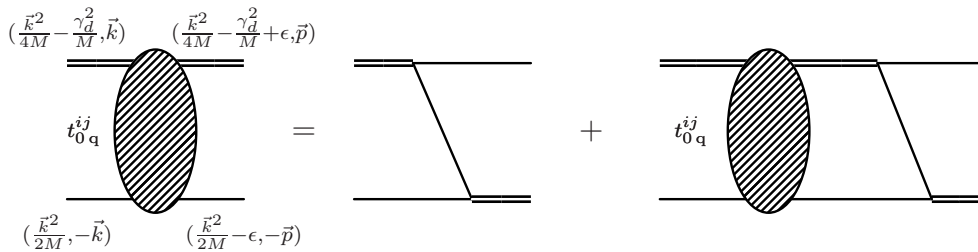


Figure 3: The Faddeev equation (3.1) which needs to be solved for  $t_{0q}^{ij}$  at LO in the quartet channel.

half off-shell amplitude at LO, let us choose the kinematics as in Fig. 3 in such a way that the incoming (outgoing) deuteron line carries momentum  $\vec{k}$  ( $\vec{p}$ ), energy  $\vec{k}^2/(4M) - \gamma_d^2/M$  ( $\vec{k}^2/(4M) - \gamma_d^2/M + \epsilon$ ) and vector index  $j$  ( $i$ ). The incoming (outgoing) nucleon line carries

momentum  $-\vec{k}$  ( $-\vec{p}$ ), energy  $\vec{k}^2/(2M)$  ( $\vec{k}^2/(2M) - \epsilon$ ) and spinor and iso-spinor indices  $\alpha$  and  $a$  ( $\beta$  and  $b$ ). Hence, the incoming deuteron and nucleon are on shell, and  $\epsilon$  denotes how far the outgoing particles are off shell. We denote by  $(it_{0q}^{ij})_{\alpha a}^{\beta b}(\vec{k}, \vec{p}, \epsilon)$  the sum of those diagrams with the kinematics above forming the half off-shell amplitude in the quartet channel, and read off from the lower line of Fig. 3 the integral equation:

$$\begin{aligned}
(it_{0q}^{ij})_{\alpha a}^{\beta b}(\vec{k}, \vec{p}, \epsilon) = & \frac{y_d^2}{2} (\sigma^j \sigma^i)_\alpha^\beta \delta_a^b \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + \epsilon - \frac{(\vec{k}+\vec{p})^2}{2M} + i\epsilon} + \\
& + \frac{y_d^2}{2} (\sigma^j \sigma^k)_\gamma^\beta \delta_c^b \int \frac{d^4 q}{(2\pi)^4} (it_{0q}^{ik})_{\alpha a}^{\gamma c}(\vec{k}, \vec{q}, \epsilon + q_0) \times \\
& \times i\Delta_d(\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + \epsilon + q_0, \vec{q}) \frac{i}{\frac{\vec{k}^2}{2M} - \epsilon - q_0 - \frac{\vec{q}^2}{2M} + i\epsilon} \times \\
& \times \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + 2\epsilon + q_0 - \frac{(\vec{q}+\vec{p})^2}{2M} + i\epsilon} .
\end{aligned} \tag{3.1}$$

The integration over  $q_0$  picks the pole at  $q_0 = (\vec{k}^2 - \vec{q}^2)/(2M) - \epsilon + i\epsilon$ . After that, we set  $\epsilon = (\vec{k}^2 - \vec{p}^2)/(2M)$ , integrate over the angle between  $\vec{k}$  and  $\vec{p}$  weighted with the Legendre polynomial of the first kind  $P_l(\hat{k} \cdot \hat{p})$  to project onto the  $l$ th partial wave and set  $i = (1 + i2)/\sqrt{2}$ ,  $j = (1 - i2)/\sqrt{2}$ ,  $\alpha = \beta = 1$ ,  $a = b = 1$ , to pick up the spin quartet part. Denoting this projected amplitude by  $it_{0q}^{(l)}(k, p)$ , we find

$$\begin{aligned}
t_{0q}^{(l)}(k, p) = & -\frac{y_d^2 M}{pk} Q_l \left( \frac{p^2 + k^2 - ME - i\epsilon}{pk} \right) - \\
& - \frac{2}{\pi} \int_0^\infty dq q^2 t_{0q}^{(l)}(k, q) \frac{1}{\sqrt{\frac{3q^2}{4} - ME - i\epsilon - \gamma_d}} \frac{1}{qp} Q_l \left( \frac{p^2 + q^2 - ME - i\epsilon}{pq} \right),
\end{aligned} \tag{3.2}$$

where  $E = 3k^2/(4M) - \gamma_d^2/M$  is the total energy and the Legendre polynomials of the second kind  $Q_l$  are defined as

$$Q_l(a) = \frac{1}{2} \int_{-1}^1 dx \frac{P_l(x)}{x + a} . \tag{3.3}$$

This Faddeev equation was first derived in [23]. Notice that when  $p = k$ , all external legs are on-shell. Although the pole on the real axis due to the deuteron propagator is regulated by the  $i\epsilon$  prescription and the logarithmic singularities occurring above threshold are integrable for each partial wave, both cause numerical instabilities. We used the Hetherington-Schick [24, 25, 26] method to numerically solve (3.2). The basic idea is to perform a rotation of the variable  $q$  into the complex plane by an angle large enough in order to avoid the singularities in and near the real axis but small enough not to cross the singularities of the kernel or of the solution. One can show that the singularities of the solution are not closer to the real axis than those of the inhomogeneous, Born term [27]. The solution

on the real axis can then be obtained from the solution on the deformed contour by using (3.2) again, now with  $k$  and  $p$  on the real axis and  $q$  on the contour. The computational effort becomes trivial, and a code runs within seconds on a personal computer.

To obtain the neutron-deuteron scattering amplitude, we have to multiply the on-shell amplitude  $t_{0q}^{(l)}(k, p)$  by the wave function renormalisation constant,

$$\begin{aligned} T_{0q}^{(l)}(k) &= \sqrt{Z_0} t_{0q}^{(l)}(k, k) \sqrt{Z_0} , \\ \frac{1}{Z_0} &= \left. i \frac{\partial}{\partial p_0} \frac{1}{i\Delta_d(p)} \right|_{p_0 = -\frac{\gamma_d^2}{M}, \vec{p}=0} \\ &= \frac{M^2 y_d^2}{8\pi \gamma_d} . \end{aligned} \quad (3.4)$$

In contradistinction to  $it_{0q}^{(l)}(k, p)$ , the scattering amplitude  $iT_{0q}^{(l)}(k)$  depends on the parameters  $y_d$  and  $\Delta_d^{(-1)}$  only through the observable  $\gamma_d$ .

At NLO, we have additional contributions only from modifications of the deuteron, as Sect. 2.2 demonstrated, namely the deuteron kinetic energy and  $\Delta_d^{(0)}$  insertions depicted in Fig. 4. One might note that the diagrams are all finite thanks to the re-formulation of

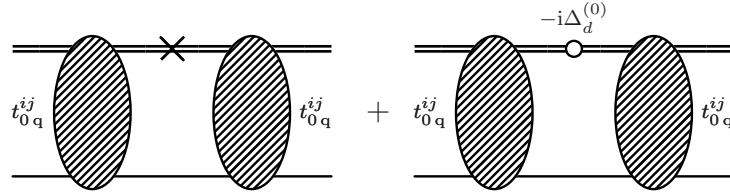


Figure 4: The NLO contributions to  $nd$  scattering in the quartet channel. Notation as in Fig. 2.

the original Lagrangean (2.1) into (2.4) and hence can be treated numerically [16]. Each contribution is inserted only once. Because of the absence of interactions which mix different partial waves, the NLO contributions are given by

$$\begin{aligned} it_{1q}^{(l)}(k) &= \int \frac{d^4 q}{(2\pi)^4} (it_{0q}^{(l)}(k, q))^2 \frac{i}{\frac{k^2}{2M} - q_0 - \frac{q^2}{2M} + i\epsilon} \times \\ &\times \left[ i\Delta_d\left(\frac{k^2}{4M} - \frac{\gamma_d^2}{M} + q_0, \vec{q}\right) \right]^2 i\mathcal{I}_d\left(\frac{k^2}{4M} - \frac{\gamma_d^2}{M} + q_0, \vec{q}\right) , \end{aligned} \quad (3.5)$$

where the corrections to the deuteron are

$$i\mathcal{I}_d(p_0, \vec{p}) = -i\Delta_d^{(0)} - i\left(p_0 - \frac{\vec{p}^2}{4M}\right) . \quad (3.6)$$

The  $q_0$  integration picks the nucleon pole resulting in a one dimensional integral

$$it_{1q}^{(l)}(k) = \int_0^\infty \frac{dq}{2\pi^2} q^2 (it_{0q}^{(l)}(k, q))^2 \left[ i\Delta_d\left(\frac{3k^2}{4M} - \frac{\gamma_d^2}{M} - \frac{q^2}{2M}, \vec{q}\right) \right]^2 \times \quad (3.7)$$

$$\times \text{i}\mathcal{I}_d\left(\frac{3k^2}{4M} - \frac{\gamma_d^2}{M} - \frac{q^2}{2M}, \vec{q}\right) ,$$

which has to be performed numerically since  $t_{0q}^{(l)}(k, p)$  is known only numerically.

Finally, the wave function renormalisation constant  $Z$  at NLO is found from

$$\begin{aligned} \frac{1}{Z} &= \frac{1}{Z_0 + Z_1} \simeq \frac{1}{Z_0} - \frac{Z_1}{Z_0^2} \\ &= \text{i} \frac{\partial}{\partial p_0} \frac{1}{\text{i}\Delta_d(p) + \text{i}\Delta_d(p)\text{i}\mathcal{I}_d(p)\text{i}\Delta_d(p)} \Big|_{p_0=\frac{\gamma_d^2}{M}, \vec{p}=0} \simeq \frac{1}{Z_0} - \text{i} \frac{\partial}{\partial p_0} \text{i}\mathcal{I}_d(p_0, \vec{p}) \Big|_{p_0=\frac{\gamma_d^2}{M}, \vec{p}=0} \end{aligned} \quad (3.8)$$

as

$$Z_1 = Z_0^2 . \quad (3.9)$$

The NLO amplitude of the  $l$ th partial wave in the quartet channel is therefore given by

$$\begin{aligned} T_q^{(l)}(k) &= Z t_q^{(l)}(k, k) \\ &\simeq (Z_0 + Z_1)(t_{0q}^{(l)}(k, k) + t_{1q}^{(l)}(k, k)) \simeq \\ &\simeq T_{0q}^{(l)}(k) + Z_0 t_{1q}^{(l)}(k, k) + Z_1 t_{0q}^{(l)}(k, k) = T_{0q}^{(l)}(k) + T_{1q}^{(l)}(k) . \end{aligned} \quad (3.10)$$

The phase shift for each partial wave is extracted by expanding both sides of the relation

$$T_q^{(l)}(k) \simeq T_{0q}^{(l)}(k) + T_{1q}^{(l)}(k) = \frac{3\pi}{M} \frac{1}{k \cot \delta^{(l)}(k) - \text{i}k} \quad (3.11)$$

in  $Q$  with  $\delta^{(l)} = \delta^{(0)(l)} + \delta^{(1)(l)} + \dots$  and keeping only linear terms,

$$\begin{aligned} \delta^{(0)(l)} &= \frac{1}{2\text{i}} \ln \left( 1 + \frac{2\text{i}kM}{3\pi} T_{0q}^{(l)}(k) \right) , \\ \delta^{(1)(l)} &= \frac{kM}{3\pi} \frac{T_{1q}^{(l)}(k)}{\left( 1 + \frac{2\text{i}kM}{3\pi} T_{0q}^{(l)}(k) \right)} . \end{aligned} \quad (3.12)$$

At NNLO, there are two kinds of contributions. One comes from the same deuteron kinetic energy and  $\Delta^{(0)}$  terms appearing at NLO, but now inserted twice. The other comes from inserting an operator describing the tensor force which appears for example in the spin triplet channel of the Lagrangean (2.1) as

$$\mathcal{L}_{NN,d}^{\text{SD}} = \frac{1}{4} C_{2d}^{(\text{SD})} (N^T P^i N)^\dagger \left[ N^T \left( P_d^j (\vec{\partial} - \vec{\partial})^i (\vec{\partial} - \vec{\partial})^j - \frac{1}{3} P_d^i (\vec{\partial} - \vec{\partial})^2 \right) N \right] + \text{H.c.} \quad (3.13)$$

and is generated by the term

$$\mathcal{L}_{Nd}^{\text{SD}} = y_d^{\text{SD}} d^{i\dagger} \left[ N \left( (\vec{\partial} - \vec{\partial})^i (\vec{\partial} - \vec{\partial})^j - \frac{1}{3} \delta^{ij} (\vec{\partial} - \vec{\partial})^2 \right) P_d^j N \right] + \text{H.c.} \quad (3.14)$$

in the Lagrangean (2.4) after the Gaußian integration over the deuteron field is performed. This operator mixes spin and angular momentum and hence produces a splitting and mixing of amplitudes with the same spin  $S$  and angular momentum  $L$  but different values of the total angular momentum  $J$ . This splitting is very important in determining spin observables. However, although our calculation for the phase shifts themselves is at NNLO and should be precise, the splittings start only at this order and should not be very realistic. We postpone the analysis of these splittings to a future publication where the spin observables will also be addressed and limit ourselves here to the phase shifts averaged over different values of  $J$ . At NNLO the tensor force appears only linearly, and a somewhat lengthy but straightforward calculation shows that, after this average, its contribution vanishes.

The following trick allows for a very simple calculation of the double insertion of the kinetic and  $\Delta^{(0)}$  operators. If they were included to all orders, the only change from the LO calculation would be the use of the NNLO deuteron propagator (2.18) in (3.1) instead of the LO one, (2.9). This procedure re-sums some contributions of order N3LO and higher, but not all of them, so it can be justified only up to NNLO. Therefore, a spin-averaged NNLO calculation is simply performed by replacing the LO deuteron propagator (2.9) by the propagator (2.18) which summarises all effective range effects, in the integral equation (3.2). As the resulting amplitude is unitary, (3.11) is used directly to extract the phase shift. This procedure has already been used in the calculation of the S wave quartet scattering in [16].

### 3.2 Doublet Channel

The calculation for the spin 1/2 channels proceeds in an analogous way to the one in the spin 3/2 channels discussed above. The main difference is that the spin zero di-baryon field  $t$  also contributes in intermediate states of  $nd$  amplitudes. The analogue of (3.1) is then a system of two coupled integral equations for the  $Nd \rightarrow Nd$  amplitude  $it_{0d,Nd \rightarrow Nd}^{ij}(\vec{k}, \vec{p}, \epsilon)$  and for the  $Nd \rightarrow Nt$  amplitude  $it_{0d,Nd \rightarrow Nt}^{iA}(\vec{k}, \vec{p}, \epsilon)$ , pictorially represented in Fig. 5:

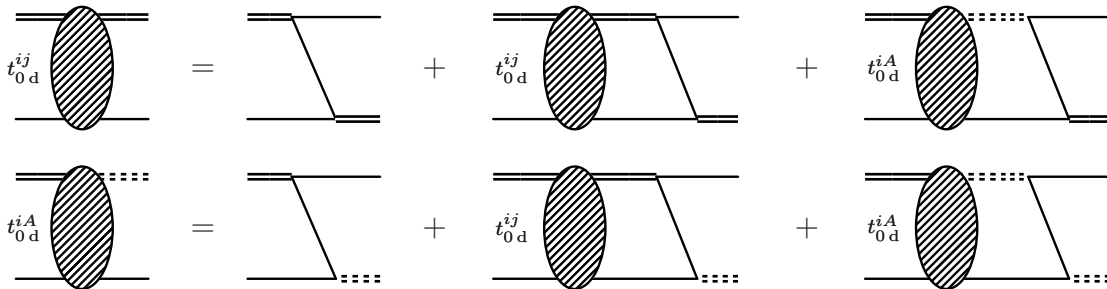


Figure 5: The coupled set of Faddeev equations (3.15/3.16) which need to be solved for  $t_{0d}$  at LO in the doublet channel. The double dashed line denotes the di-baryon field  $t^A$ .

$$\begin{aligned}
(\text{it}_{0\text{d}, Nd \rightarrow Nd}^{ij})_{\alpha a}^{\beta b}(\vec{k}, \vec{p}, \epsilon) &= \frac{y_d^2}{2} (\sigma^j \sigma^i)_\alpha^\beta \delta_a^b \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + \epsilon - \frac{(\vec{k}+\vec{p})^2}{2M} + i\epsilon} + \\
&+ \int \frac{d^4 q}{(2\pi)^4} \left[ 4y_d^2 (\sigma^j \sigma^k)_\gamma^\beta \delta_c^b (\text{it}_{0\text{d}, Nd \rightarrow Nd}^{ik})_{\alpha a}^{\gamma c}(\vec{k}, \vec{q}, \epsilon + q_0) \times \right. \\
&\quad \times i\Delta_d(\frac{\vec{k}^2}{4} - \frac{\gamma_d^2}{M} + \epsilon + q_0, \vec{q}) + \\
&\quad + 4y_d y_t (\sigma^j)_\gamma^\beta (\tau^C)_c^b (\text{it}_{0\text{d}, Nd \rightarrow Nt}^{iC})_{\alpha a}^{\gamma c}(\vec{k}, \vec{q}, \epsilon + q_0) \times \\
&\quad \times i\Delta_t(\frac{\vec{k}^2}{4} - \frac{\gamma_d^2}{M} + \epsilon + q_0, \vec{q}) \left. \right] \times \\
&\quad \times \frac{i}{\frac{\vec{k}^2}{2M} - \epsilon - q_0 - \frac{\vec{q}^2}{2M} + i\epsilon} \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + 2\epsilon + q_0 - \frac{(\vec{q}+\vec{p})^2}{2M} + i\epsilon}
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
(\text{it}_{0\text{d}, Nd \rightarrow Nt}^{iA})_{\alpha a}^{\beta b}(\vec{k}, \vec{p}, \epsilon) &= \frac{y_d y_t}{2} (\sigma^i)_\alpha^\beta (\tau^A)_a^b \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + \epsilon - \frac{(\vec{k}+\vec{p})^2}{2M} + i\epsilon} + \\
&+ \int \frac{d^4 q}{(2\pi)^4} \left[ 4y_t^2 \delta_\gamma^\beta (\tau^A \tau^C)_c^b (\text{it}_{0\text{d}, Nd \rightarrow Nt}^{iC})_{\alpha a}^{\gamma c}(\vec{k}, \vec{q}, \epsilon + q_0) \times \right. \\
&\quad \times i\Delta_t(\frac{\vec{k}^2}{4} - \frac{\gamma_d^2}{M} + \epsilon + q_0, \vec{q}) + \\
&\quad + 4y_d y_t (\sigma^k)_\gamma^\beta (\tau^A)_c^b (\text{it}_{0\text{d}, Nd \rightarrow Nd}^{ik})_{\alpha a}^{\gamma c}(\vec{k}, \vec{q}, \epsilon + q_0) \times \\
&\quad \times i\Delta_d(\frac{\vec{k}^2}{4} - \frac{\gamma_d^2}{M} + \epsilon + q_0, \vec{q}) \left. \right] \times \\
&\quad \times \frac{i}{\frac{\vec{k}^2}{2M} - \epsilon - q_0 - \frac{\vec{q}^2}{2M} + i\epsilon} \frac{i}{-\frac{\vec{k}^2}{4M} - \frac{\gamma_d^2}{M} + 2\epsilon + q_0 - \frac{(\vec{q}+\vec{p})^2}{2M} + i\epsilon}
\end{aligned} \tag{3.16}$$

After projecting on the spin 1/2 channel using

$$\begin{aligned}
(t_{0\text{d}, Nd \rightarrow Nd})_{\alpha a}^{\beta b} &= \frac{1}{3} (\sigma^i)_\alpha^{\alpha'} (\text{it}_{0\text{d}, Nd \rightarrow Nd}^{ij})_{\alpha' a}^{\beta' b} (\sigma^j)_{\beta'}^\beta, \\
(t_{0\text{d}, Nd \rightarrow Nt})_{\alpha a}^{\beta b} &= \frac{1}{3} (\sigma^i)_\alpha^{\alpha'} (\text{it}_{0\text{d}, Nd \rightarrow Nt}^{iA})_{\alpha' a}^{\beta' b} (\tau^A)_{\beta'}^b,
\end{aligned} \tag{3.17}$$

and following the same steps as in the quartet case one finds

$$\begin{aligned}
t_{0\text{d}, Nd \rightarrow Nd}^{(l)}(k, p) &= \frac{4y_d^2 M}{pk} Q_l \left( \frac{p^2 + k^2 - ME - i\epsilon}{pk} \right) + \\
&+ \frac{1}{\pi} \int_0^\infty dq q^2 t_{0\text{d}, Nd \rightarrow Nd}^{(l)}(k, q) \frac{1}{\sqrt{\frac{3q^2}{4} - ME - i\epsilon - \gamma_d}} \times
\end{aligned}$$

$$\begin{aligned}
& \times \frac{1}{qp} Q_l \left( \frac{p^2 + q^2 - ME - i\varepsilon}{pq} \right) - \\
& - \frac{3}{\pi} \frac{y_d}{y_t} \int_0^\infty dq q^2 t_{0d, Nd \rightarrow Nt}^{(l)}(k, q) \frac{1}{\sqrt{\frac{3q^2}{4} - ME - i\varepsilon} - \gamma_t} \times \\
& \times \frac{1}{qp} Q_l \left( \frac{p^2 + q^2 - ME - i\varepsilon}{pq} \right) ,
\end{aligned} \tag{3.18}$$

$$\begin{aligned}
t_{0d, Nd \rightarrow Nt}^{(l)}(k, p) &= - \frac{12y_d y_t M}{pk} Q_l \left( \frac{p^2 + k^2 - ME - i\varepsilon}{pk} \right) + \\
& + \frac{1}{\pi} \int_0^\infty dq q^2 t_{0d, Nd \rightarrow Nt}^{(l)}(k, q) \frac{1}{\sqrt{\frac{3q^2}{4} - ME - i\varepsilon} - \gamma_t} \times \\
& \times \frac{1}{qp} Q_l \left( \frac{p^2 + q^2 - ME - i\varepsilon}{pq} \right) - \\
& - \frac{3}{\pi} \frac{y_t}{y_d} \int_0^\infty dq q^2 t_{0d, Nd \rightarrow Nd}^{(l)}(k, q) \frac{1}{\sqrt{\frac{3q^2}{4} - ME - i\varepsilon} - \gamma_d} \times \\
& \times \frac{1}{qp} Q_l \left( \frac{p^2 + q^2 - ME - i\varepsilon}{pq} \right) .
\end{aligned} \tag{3.19}$$

The modifications for the NLO and NNLO calculation are straightforward. Fig. 6 shows the graphs giving the NLO corrections. At NNLO, the propagators  $i\Delta_{d/t}(p)$  in the above integral equations are replaced by  $i\Delta_{d/t}(p; \text{N3LO})$  of (2.18).

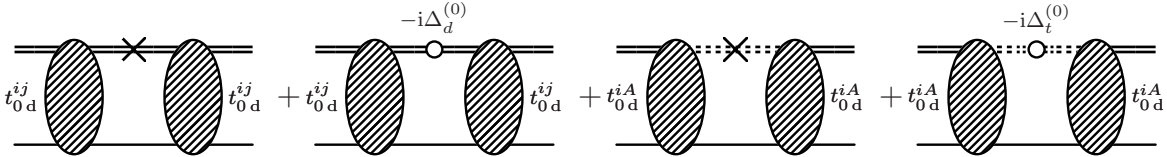


Figure 6: The NLO contributions to  $nd$  scattering in the doublet channel.

## 4 Discussion and Conclusions

We used  $\hbar c = 197.327$  MeV fm, a nucleon mass of  $M = 938.918$  MeV, for the  $NN$  triplet channel a deuteron binding energy (momentum) of  $B = 2.225$  MeV ( $\gamma_d = 45.7066$  MeV),

a residue of  $Z_d = 1.690(3)$ , and for the  $NN$  singlet channel an  $^1S_0$  scattering length of  $a_t = -23.714$  fm and effective range  $r_{0t} = 2.73$  fm. In Fig. 7, we present the results of our computations for the real and imaginary parts of the quartet partial waves  $l = 0$  to 4 in the centre-of-mass frame. Fig. 8 shows the real and imaginary parts of the doublet partial waves  $l = 1$  to 4. The  $^4S_{\frac{3}{2}}$  partial wave was already computed in a previous publication [16] with a choice of parameters and method to extract the phase shift which differs from the one used here formally only at higher order in the power counting, and indeed the difference is marginal. Comparison of the LO with the NLO and NNLO result demonstrates convergence of the effective field theory, with the expansion parameter found to be about  $\frac{1}{3}$ . We therefore claim that the NNLO calculation has an error bar of less than 4 percent. It is interesting to note that the NLO correction is for high enough energies sometimes sizeable, while the NNLO correction is in general very small. In contradistinction to the  $NN$  case, the hierarchy the phase shifts exhibit at large momenta with increasing  $l$  is not due to suppression of the higher partial waves in powers of  $Q$ , as the diagrams scale for all partial waves in the same way. Rather, it is easy to see that especially for the quartet channel, already the behaviour of the LO integral equations (3.2) and (3.18/3.19) as  $p = k \gg \gamma_{d/t}$  makes the  $l$ th partial wave asymptotically go like the  $l$ th Legendre polynomial of the second kind  $Q_l(\frac{5}{4})$ , and  $Q_l(\frac{5}{4})/Q_{l-1}(\frac{5}{4}) \approx 0.33 - 0.45$ , which is by accident close to the expansion parameter  $Q$ .

Below the deuteron breakup point, our calculation is in good agreement with the variational calculation of Kievsky et al. [28] which is based on the AV18 potential. We also compare to partial wave analyses of  $pd$  scattering experiments by Huttel et al. [30] and Schmelzbach et al. [31] as Coulomb and chiral symmetry breaking effects can be neglected at high momenta. Above  $E_{\text{cm}} \sim 15$  MeV, i.e. a cm-momentum of about 140 MeV, one expects the pion-less theory to diverge from experiment because not later than then should the pion manifest itself as an explicit degree of freedom of the low energy theory.

Within the range of validity of this pion-less theory we seem to have good convergence and our results agree with potential model calculations within the theoretical uncertainty of  $(1/3)^3 \approx 4\%$ . That makes us optimistic about carrying out higher order calculations of problematic spin observables where our approach will differ from potential model calculations due to the inclusion of three-body forces. For example, the nucleon-deuteron vector analysing power  $A_y$  in elastic  $Nd$  scattering at energies below  $E_{\text{lab}} \approx 30$  MeV does not have a satisfactory description in any realistic potential model of  $NN$  scattering, see [32, 33] for details. It remains to be seen whether the EFT approach can be of help there.

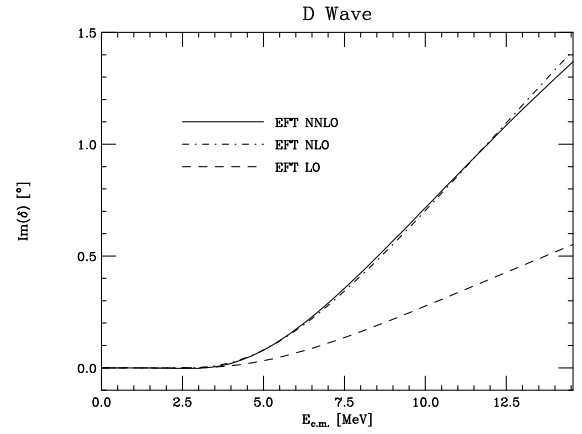
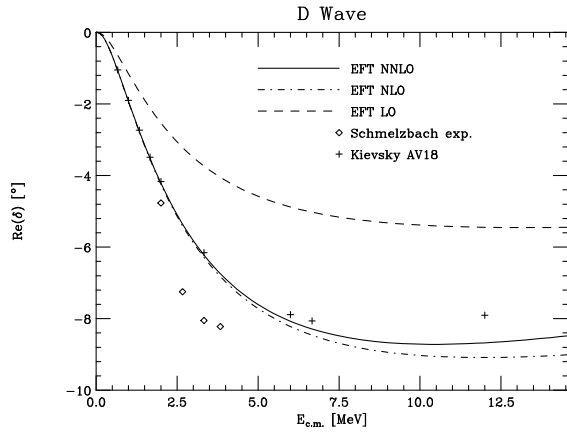
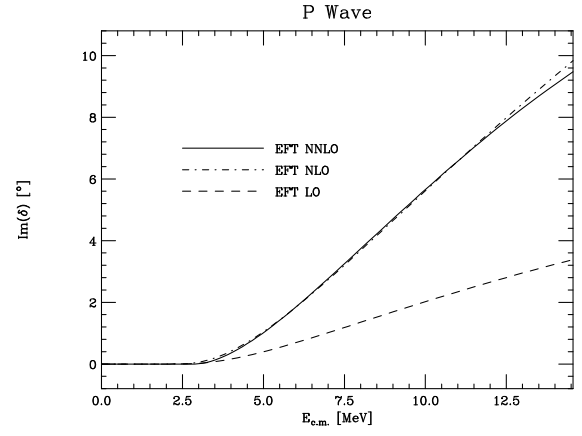
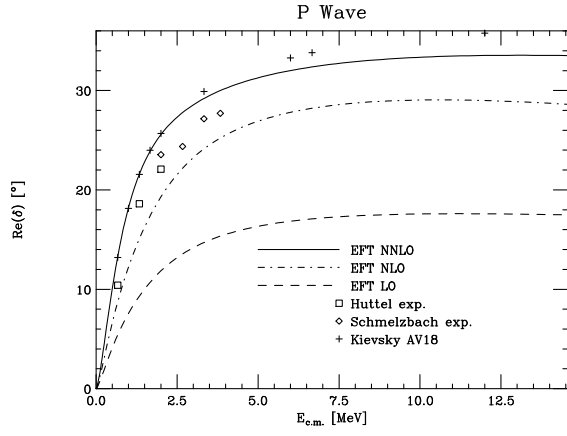
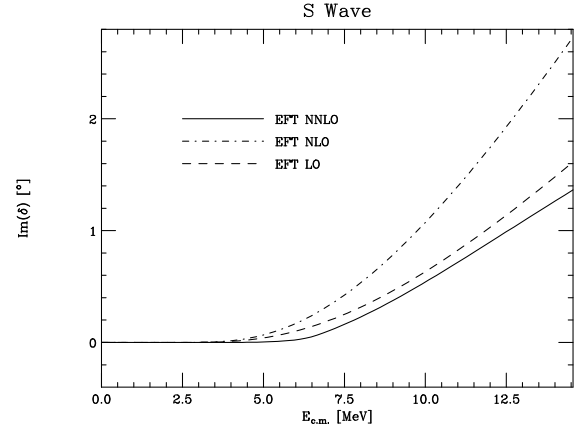
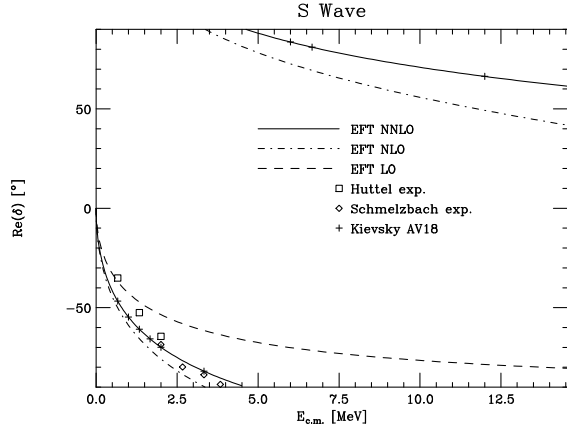
We close with a note comparing the EFT approach at this order to realistic potential model calculations. Since we use as input parameters to NNLO only the scattering lengths  $1/\gamma_d$ ,  $a_t$  and effective ranges  $\rho_d$ ,  $r_{0t}$  of the two body system in the S wave channels, we expect any realistic potential model which reproduces these numbers to agree with our results within the error bar of an NNLO calculation,  $\sim 4\%$ . The absence of counterterms at this order (e.g. of three body forces) guarantees that an effective theory of a realistic potential model will be identical to the EFT discussed here. Hence, a potential model can already serve for the purpose of comparing to our results at energies considerably higher than the deuteron breakup point. However, the EFT approach is systematic and rigorous, allowing for error estimates. The small number and simplicity of the interactions in the Lagrangean



at NNLO makes the computations considerably simpler: The LO integral equation is only one-dimensional and can be obtained in very short computing time, and at NLO, only (partially analytical, partially simple numerical) one dimensional integrations are necessary, in contradistinction to many dimensional integral equations in traditional approaches. Calculations of phase shifts in any approach, be it potential models or EFT with pions as explicit degrees of freedom, are hence bound to reproduce our results within error-bars in the range of validity of the pion-less theory, allowing for valuable cross-checks. Finally, rigorously determined three body forces will enter at higher orders of the EFT approach and lift the similarity to potential model calculations.

## Acknowledgements

We are indebted to G. Rupak, M. J. Savage and the effective field theory group at the INT and the University of Washington in Seattle for a number of valuable discussions, and to A. Kievsky for communicating his results for the higher partial waves above breakup prior to publication. F. G. thanks the Nuclear Theory Group of the University of Washington, Seattle, for its kind hospitality. The work was supported in part by the Department of Energy grants DE-FG02-96ER40945 (F.G.) and DE-FG03-97ER41014 (H.W.G.), and by the Bundesministerium für Bildung und Forschung (H.W.G.).



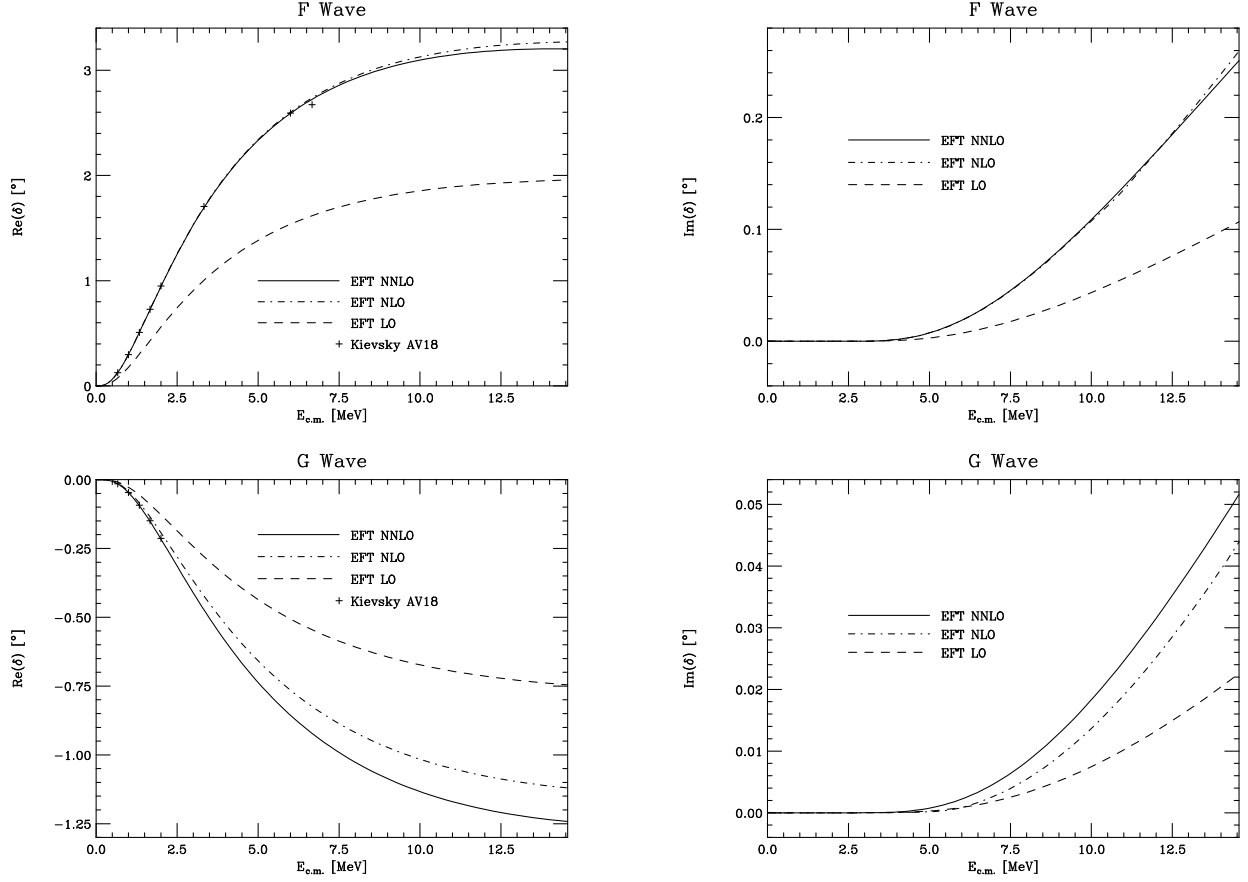
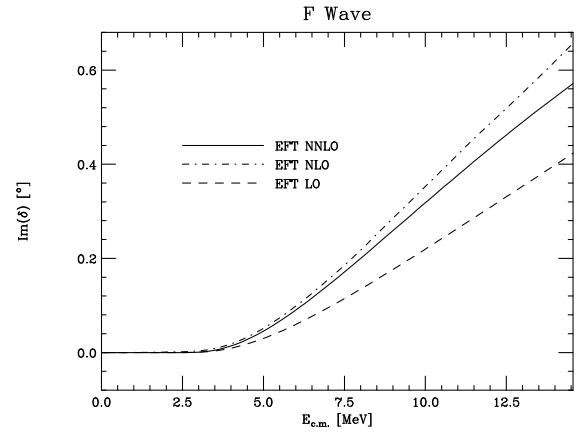
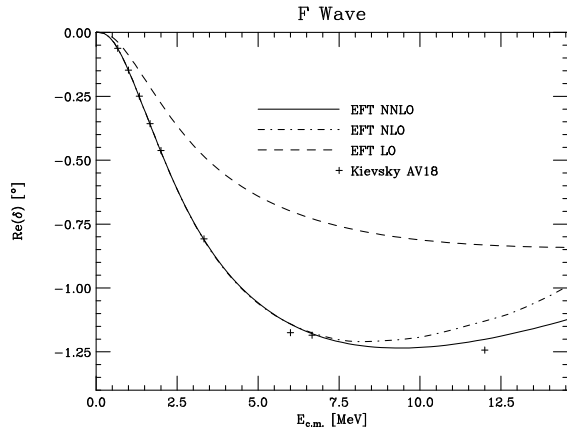
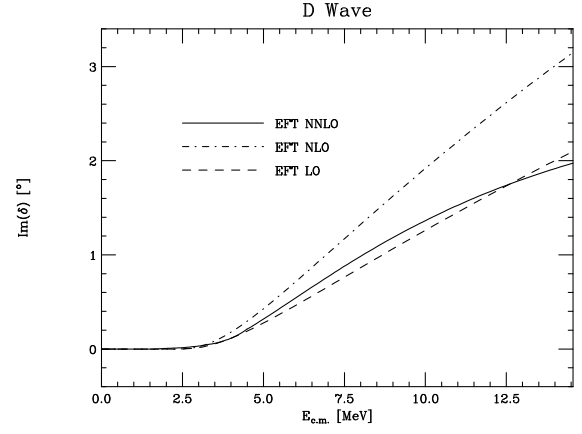
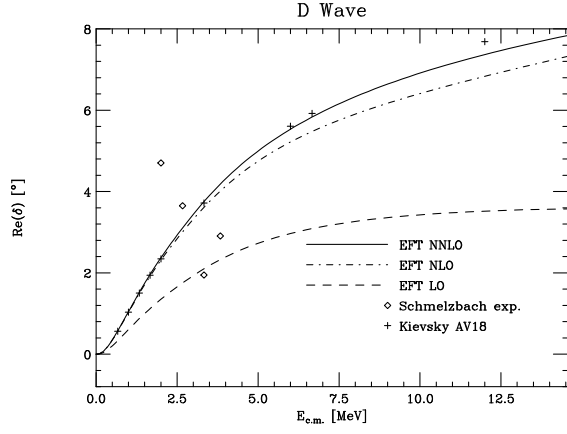
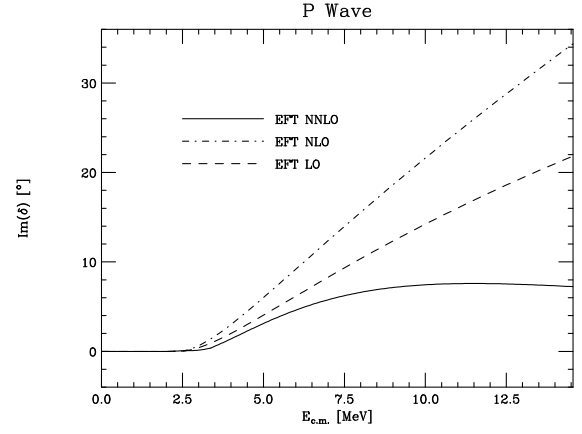
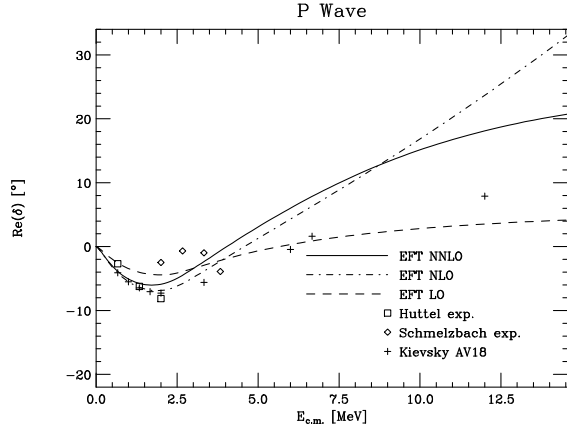


Figure 7: Real and imaginary parts of the first five partial waves in the quartet channel of  $nd$  scattering versus the centre-of-mass energy. The dashed line is the LO, the dot-dashed the NLO and the solid line the NNLO result. The calculations of Kievsky et al. (crosses) are from Refs. [28] below breakup ( $E_{\text{cm}} = B$ ) and [29] above breakup. The phase shift analyses of  $pd$  data by Huttel et al. [30] and Schmelzbach et al. [31] are presented as open squares and diamonds, respectively.



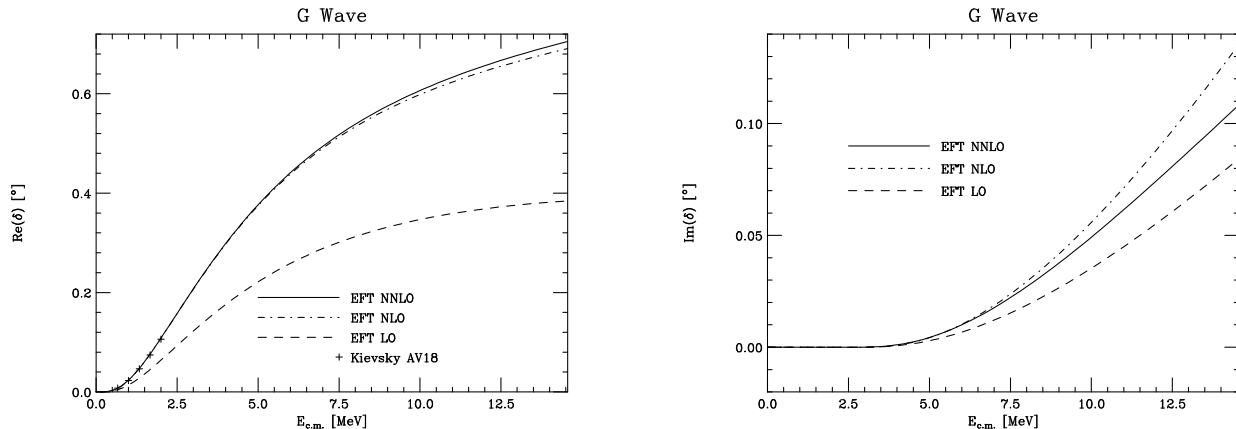


Figure 8: Real and imaginary parts of the first four higher partial waves in the doublet channel of  $nd$  scattering versus the centre-of-mass energy. Notation as in Fig. 7.

## References

- [1] G.P. Lepage, “What Is Renormalization?,” *Invited lectures given at TASI’89 Summer School, Boulder, CO, Jun 4-30, 1989*, in: From Actions to Answers, T. DeGrand and D. Toussaint, eds., World Scientific 1990.
- [2] J. Gasser and H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).
- [3] S. Weinberg, *Physica* **96A**, 327 (1979).
- [4] J. Gasser, M.E. Sainio and A. Svarc, *Nucl. Phys.* **B307**, 779 (1988).
- [5] E. Jenkins and A.V. Manohar, *Phys. Lett.* **B255**, 558 (1991).
- [6] D.B. Kaplan, M.J. Savage and M.B. Wise, *Phys. Rev.* **C59**, 617 (1999), nucl-th/9804032.
- [7] J. Chen, H.W. Griesshammer, M.J. Savage and R.P. Springer, *Nucl. Phys.* **A644**, 221 (1998), nucl-th/9806080.
- [8] J. Chen, H.W. Griesshammer, M.J. Savage and R.P. Springer, *Nucl. Phys.* **A644**, 245 (1998), nucl-th/9809023.
- [9] D.B. Kaplan, M.J. Savage, R.P. Springer and M.B. Wise, *Phys. Lett.* **B449**, 1 (1999), nucl-th/9807081.
- [10] M.J. Savage, K.A. Scaldeferri and M.B. Wise, *Nucl. Phys.* **A652**, 273 (1999), nucl-th/9811029.

- [11] T. Mehen and I.W. Stewart, nucl-th/9906010, to appear in the proceedings of Workshop on Nuclear Physics with Effective Field Theory: 1999, INT Seattle, Feb. 25-26, 1999.; S. Fleming, T. Mehen and I.W. Stewart, nucl-th/9906056.
- [12] M.J. Savage and R.P. Springer, *Nucl. Phys.* **A644**, 235 (1998), nucl-th/9807014.
- [13] E. Epelbaum and Ulf-G. Meißner, *Phys. Lett.* **B461**, 287 (1999), nucl-th/9902042;  
E. Epelbaum and Ulf-G. Meißner, nucl-th/9903046, to appear in the proceedings of Workshop on Nuclear Physics with Effective Field Theory: 1999, INT Seattle, Feb. 25-26, 1999.
- [14] S. Weinberg, *Nucl. Phys.* **B363**, 3 (1991).
- [15] J. Chen, G. Rupak and M.J. Savage, *Nucl. Phys.* **A653**, 386 (1999), nucl-th/9902056.
- [16] P.F. Bedaque and H.W. Griesshammer, nucl-th/9907077 (to appear in *Nucl. Phys. A*).
- [17] P.F. Bedaque, H.W. Hammer and U. van Kolck, *Phys. Rev. Lett.* **82**, 463 (1999), nucl-th/9809025;  
P.F. Bedaque, H.W. Hammer and U. van Kolck, *Nucl. Phys.* **A646**, 444 (1999), nucl-th/9811046;  
P.F. Bedaque, H.W. Hammer and U. van Kolck, nucl-th/9906032 (to appear in *Phys. Rev. C*).
- [18] M.C. Birse, J.A. McGovern and K.A. Richardson, *Phys. Lett.* **B464**, 169 (1999), hep-ph/9807302.
- [19] U. van Kolck, *Nucl. Phys.* **A645**, 273 (1999), nucl-th/9808007.
- [20] D.B. Kaplan, M.J. Savage and M.B. Wise, *Phys. Lett.* **B424**, 390 (1998), nucl-th/9801034;  
D.B. Kaplan, M.J. Savage and M.B. Wise, *Nucl. Phys.* **B534**, 329 (1998), nucl-th/9802075.
- [21] G. Rupak and N. Shores, *Phys. Rev.* **C60**, 054004 (1999), nucl-th/9902077.
- [22] D.R. Phillips, G. Rupak and M.J. Savage, nucl-th/9908054.
- [23] G.V. Skornyakov and K.A. Ter-Martirosian, *Sov. Phys. JETP* **4** (1957) 648.
- [24] J.H. Hetherington and L.H. Schick, *Phys. Rev.* **B137**, 935 (1965);  
R.T. Cahill and I.H. Sloan, *Nucl. Phys.* **A165**, 161 (1971).
- [25] R. Aaron and R.D. Amado, *Phys. Rev.* **150**, 857 (1966).

- [26] E.W. Schmid and H. Ziegelmann, The Quantum Mechanical Three-Body Problem, Vieweg Tracts in Pure and Applied Physics Vol. 2, Pergamon Press (1974).
- [27] D.D. Brayshaw, *Phys. Rev.* **176**, 1855 (1968).
- [28] A. Kievsky, S. Rosati, W. Tornow and M. Viviani, *Nucl. Phys.* **A607**, 402 (1996).
- [29] A. Kievsky, private communication.
- [30] E. Huttel, W. Arnold, H. Baumgart, H. Berg and G. Clausnitzer, *Nucl. Phys.* **A406**, 443 (1983).
- [31] P.A. Schmelzbach, W. Gröbler, R.E. White, V. König, R. Risler and P. Marmier, *Nucl. Phys.* **A197**, 273 (1972).
- [32] D. Hüber and J.L. Friar, *Phys. Rev.* **C58**, 674 (1998), nucl-th/9803038.
- [33] A. Kievsky, nucl-th/9905045 (to appear in *Phys. Rev. C*).